

Selected Items in Jet Algorithms

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Summary. — I provide a very brief overview of recent developments in jet algorithms, mostly focusing on the issue of infrared-safety.

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1. – Introduction

The main properties that should be satisfied by any jet definition were already pointed out almost twenty years ago [1, 2]: the jet algorithm should be 1) simple to implement both in experimental analysis and theoretical calculations, 2) well defined and yielding a finite cross section at any order of perturbation theory, 3) relatively insensitive to hadronization. Many different jet definitions have been proposed in recent years but it turns out that some of them do not strictly meet the above features. This could in principle lead to serious problems especially when infrared(IR)-safety (i.e., the second item in the above list) is not correctly implemented, since in this case the matching with fixed-order theoretical results would be spoiled and the whole jet-finding procedure would heavily depend on non-perturbative effects (hadronization, underlying event, pile-up). We can group jet algorithms in two broad classes: Iterative Cone (IC) and Sequential Recombination (SR). IC algorithms cluster particles according to their relative distance in *coordinate-space* and have been extensively used at past lepton and hadron colliders. SR algorithms cluster particles according to their relative distance in *momentum-space* and are somehow preferred by theorists since they rigorously take into account IR-safety. In what follows I will try to give an overview of recent developments obtained in both the IC and SR classes. For a complete and extensive treatment of these and other aspects of jet algorithms I refer the reader to the recent literature on the subject [3, 4].

2. – Iterative-Cone algorithms

The core structure of any IC algorithm [5] can be described as follows: choose a *seed* particle i , sum the momenta of all particles j within a cone of radius R (in y and ϕ) around i , take the direction of this sum as a new seed, and repeat until the cone

is stable (i.e., the direction of the n -th seed coincides with the direction of the $(n-1)$ -th). This procedure, however, may eventually lead to find multiple stable cones sharing the same particles, i.e. the cones may overlap. The problem can be solved either by preventing the overlap (Progressive Removal (IC-PR) algorithms) or through a splitting procedure (Split-Merge (IC-SM) algorithms). The IC-PR algorithm starts the iteration from the particle with largest- p_T and, once a stable cone is found, it removes all particles inside it. Then the procedure starts again with the hardest remaining particle and go on until no particles are left. This algorithm is also known as UA1-cone [6], since it was first introduced and extensively used by the CERN UA1 collaboration. It is quite easy to see that this algorithm is IR(collinear)-unsafe. Assume that the hardest particle undergoes a collinear splitting $p_1 \rightarrow p_{1a}, p_{1b}$ with $p_{T,1a}, p_{T,1b} < p_{T,2} < p_{T,1}$: in this case the IC-PR algorithm would lead to a different number/configuration of jets, since the p_T -ordering has been modified by the collinear emission. The IC-SM algorithm does not rely on any particular ordering instead. Once all stable cones have been found the prescription is to *merge* a pair of cones if more than a fraction f (typically $f=0.5-0.7$) of the softer's cone p_T is in common with the harder one, or to *assign* the shared particles to the closer cone. The *split-merge* procedure is repeated until there are no overlapping cones left. Unfortunately the IC-SM algorithm is IR(soft)-unsafe as well [7]. Assume that two stable cones are generated starting from two hard partons whose relative distance is between R and $2R$: the addition of an extra soft particle in between would act as a new seed and the third stable cone would be found, again leading to a different number/configuration of jets. A partial solution to this problem was provided by the *Midpoint Algorithm* [8]: after all possible jets have been found, run the algorithm again using additional *midpoint* seeds between each pair of stable cones. This prescription fixes the IR(soft) issue of the IC-SM algorithm, since the result is now not dependent on the presence of an extra soft seed in the overlap region, and has been adopted as a recommendation for Run II of the Tevatron [9]. Recently [10] it has been pointed out that, for particular configurations involving more than two partons, the Midpoint algorithm is not able to find all stable cones: for exclusive quantities and/or multi-jet configurations, the midpoint prescription is still IR(soft)-unsafe. The IR issue is definitely solved by the introduction of Seedless algorithms, first proposed in [9]. The idea is to identify all possible subsets of N particles in an event and, for each subset M , check if the cone defined by the azimuth and rapidity of the total momentum of M contains other particles outside the subset: if this is not the case, then M defines a stable cone. With this prescription the jet-finding algorithm is infrared safe at all perturbative orders: the main drawback is that the clustering time ($\mathcal{O}(N \times 2^N)$) leads to extremely slow performances for $N > 4-5$. The seedless algorithm has been recently improved by the SIS-Cone (Seedless Infrared Safe Cone) implementation [10], in which the clustering time is sensibly lowered ($\mathcal{O}(N^2 \log N)$, comparable to Midpoint). The switching from the midpoint to the seedless cone is expected to have a significant impact only on exclusive quantities (i.e., jet mass distribution in multi-jet events), while the impact for inclusive observables should be modest since Midpoint IR-unsafety only appears at relatively high orders in perturbation theory.

3. – Sequential Recombination algorithms

The SR algorithm starts with the introduction of a distance $d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2}$ between particles i, j (where Δ is their distance in the y, ϕ plane and k_T their transverse-

momentum), and the distance $d_{iB} = k_{ti}^{2p}$ between the particle i and the beam. If $d_{ij} < d_{iB}$ then merge i and j , otherwise call i a jet and remove it from the iteration. There are different types of SR algorithms, depending on the value of the integer p in the definition of the distances: $p = 1$ identifies the *inclusive- k_T* algorithm [11, 12], $p = 0$ defines the *Cambridge-Aachen* algorithm [13, 14], while for $p = -1$ we have the recently proposed *anti- k_T* algorithm [15]. All the SR prescriptions are rigorously IR-safe: any soft parton is first merged with the closest hard parton and only at this point the decision about the merging of two jets is taken, depending exclusively on their opening angle. Moreover, there are no more overlap problems, since any parton is unequivocally assigned to only one jet. A very fast implementation (clustering time $\sim \mathcal{O}(N \log N)$) of all the above SR algorithms is available (FastJet [16]): it also includes an interface for the algorithms belonging to the IC class. Another public code providing access to both SR and IC algorithms is SpartyJet [17].

4. – Summary

The issue of IR-safety of a jet algorithm should be seriously taken into account, since multi-jet configurations are sensitive to it and will be far more widespread at the LHC than at previous colliders. In addition, without an IR-safe prescription, it would not be possible to fully exploit the results provided by the theoretical community involved in NLO multi-leg calculations. Several *fast* and *safe* algorithms (SIS-Cone and the SR class) are now available in public packages, but no definite advantages for a particular algorithm over the others have been found up to now: the use of different prescriptions for physics analysis and a continuous cross-checking of results is thus recommended especially for events with high jet-multiplicity at the LHC.

REFERENCES

- [1] S. D. Ellis, Z. Kunszt and D. E. Soper, Phys. Rev. D **40**, 2188 (1989).
- [2] J. E. Huth *et al.*, Fermilab-Conf-90-249-E (1990).
- [3] S. D. Ellis, J. Huston, K. Hatakeyama, P. Loch and M. Tonnesmann, Prog. Part. Nucl. Phys. **60**, 484 (2008), and references therein.
- [4] C. Buttar *et al.*, arXiv:0803.0678 [hep-ph], and references therein.
- [5] G. Arnison *et al.* [UA1 Collaboration], Phys. Lett. B **123**, 115 (1983).
- [6] G. Arnison *et al.* [UA1 Collaboration], Phys. Lett. B **132**, 214 (1983).
- [7] M. H. Seymour, Nucl. Phys. B **513**, 269 (1998).
- [8] S. D. Ellis, private communication to the OPAL Collaboration; D. E. Soper and H. C. Yang, private communication to the OPAL Collaboration; L. A. del Pozo, University of Cambridge PhD Thesis, RALT-002, (1993); R. Akers *et al.*, OPAL Collaboration, Z. Phys. C **63**, 197 (1994).
- [9] G. C. Blazey *et al.*, arXiv:hep-ex/0005012.
- [10] G. P. Salam and G. Soyez, JHEP **0705**, 086 (2007).
- [11] S. Catani, Y. L. Dokshitzer, M. H. Seymour and B. R. Webber, Nucl. Phys. B **406**, 187 (1993).
- [12] S. D. Ellis and D. E. Soper, Phys. Rev. D **48**, 3160 (1993).
- [13] Y. L. Dokshitzer, G. D. Leder, S. Moretti and B. R. Webber, JHEP **9708**, 001 (1997).
- [14] M. Wobisch and T. Wengler, arXiv:hep-ph/9907280.
- [15] M. Cacciari, G. P. Salam and G. Soyez, JHEP **0804**, 063 (2008).
- [16] M. Cacciari and G. P. Salam, Phys. Lett. B **641**, 57 (2006); <http://www.lpthe.jussieu.fr/~salam/fastjet/>
- [17] <http://www.pa.msu.edu/~huston/SpartyJet/SpartyJet.html>.